Three-Body Collisions of Binary Stars with Field Stars

Osama Elgabori, Morgan Long, Brian Stewart

Introduction

The present work seeks to gain understanding of the topology of the planar gravitational three-body problem for equal masses and to apply the results to its molecular equivalent.

The three-body problem is notorious for the complexity of its solutions that provide insight into the motion of three masses at a planetary scale or molecular scale, depending on a given potential. Even the restricted case of planar motion with equal masses that we explore yields intricate results. These results are simplified into four categories: dissociation, fly-by, exchange, and resonance events.

For modeling the problem, we utilize a program that implements the Hermite numerical integration method and reproduce previous work done by Aaron Larner to ensure that the program is accurate. We extend that work by computing the initial and final angular and radial actions for fly-by and exchange events and generate contour plots to glean further information from the gravitational case.

Computational Methods

Hermite numerical integration is used to model the restricted three-body in a C program. The conditions of the problem implemented in the program are as follows:

1. All stars have equal masses and remain in the plane..
2. Two of the stars form a binary that has a circular orbit initially.
3. The third star comes in from infinity with some impact parameter *b* and initial velocity *v* with respect to the binary center of mass.
4. All particles are under the influence of the gravitational potential.

$$U(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = -\sum_{i<j} \frac{Gm_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|} \qquad (1)$$

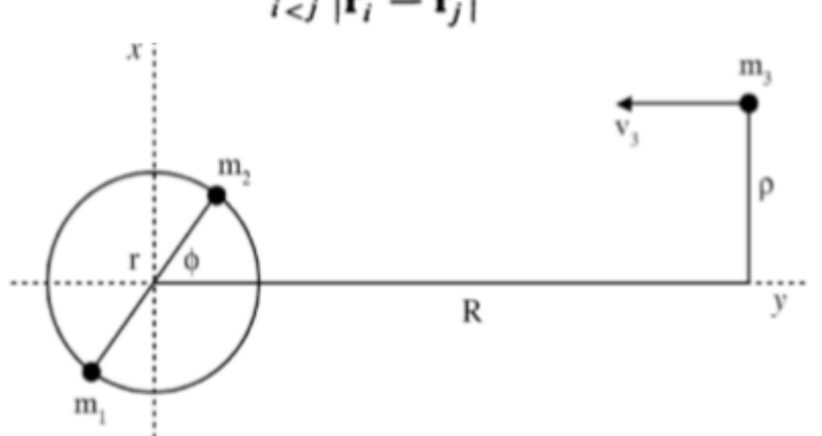

Figure 1: Schematic set up of the three-body problem adapted from H83.

Numerical Scattering Experiments

We perform one million scattering experiments for different incoming velocities. Green points represent dissociation events, red and black are exchange events with mass 2 and mass 1 respectively, white points are fly-by events, and yellow points are trajectories that failed to conserve energy. The incoming velocities chosen are multiples of the critical velocity v_c, the velocity for which total energy of the three-body system is zero. All plots are made using the same R_0 and same initial step size, except for figure 7.

Numerical Scattering Results

As we decrease the incoming velocity of the field star, fly-by events in the region encompassing the area of the binary's circular orbit, which is the impact parameter range (-1,1), become less common. This leads to more exchange and dissociation events. The decrease also causes the exchange and dissociation events to expand outside of the binary's circular region as displayed prominently in figures 4-7. At the critical velocity (Figure 6 and Figure 7), dissociation events become less common and nearly non-existent when we increase the starting distance of the field star to better approximate its approach from infinity.

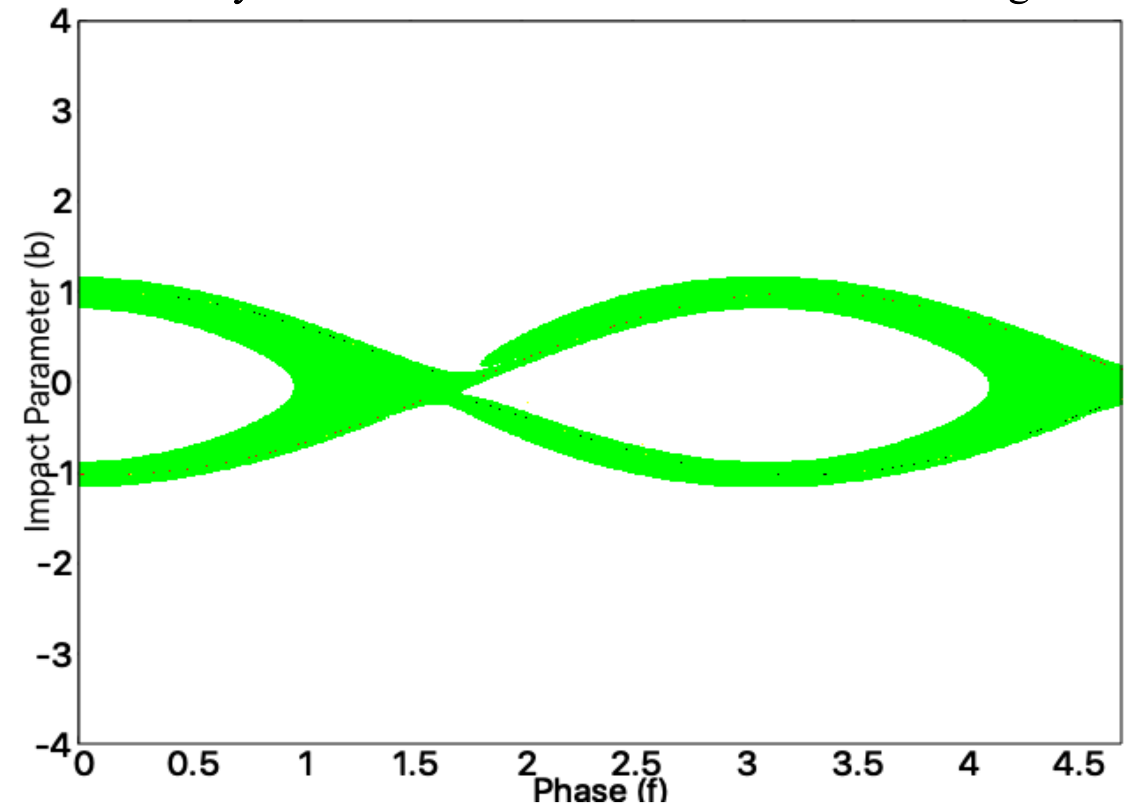

Figure 2: Scattering outcomes for $v=16v_c$, $R_0=40$, and initial step size of 0.001.

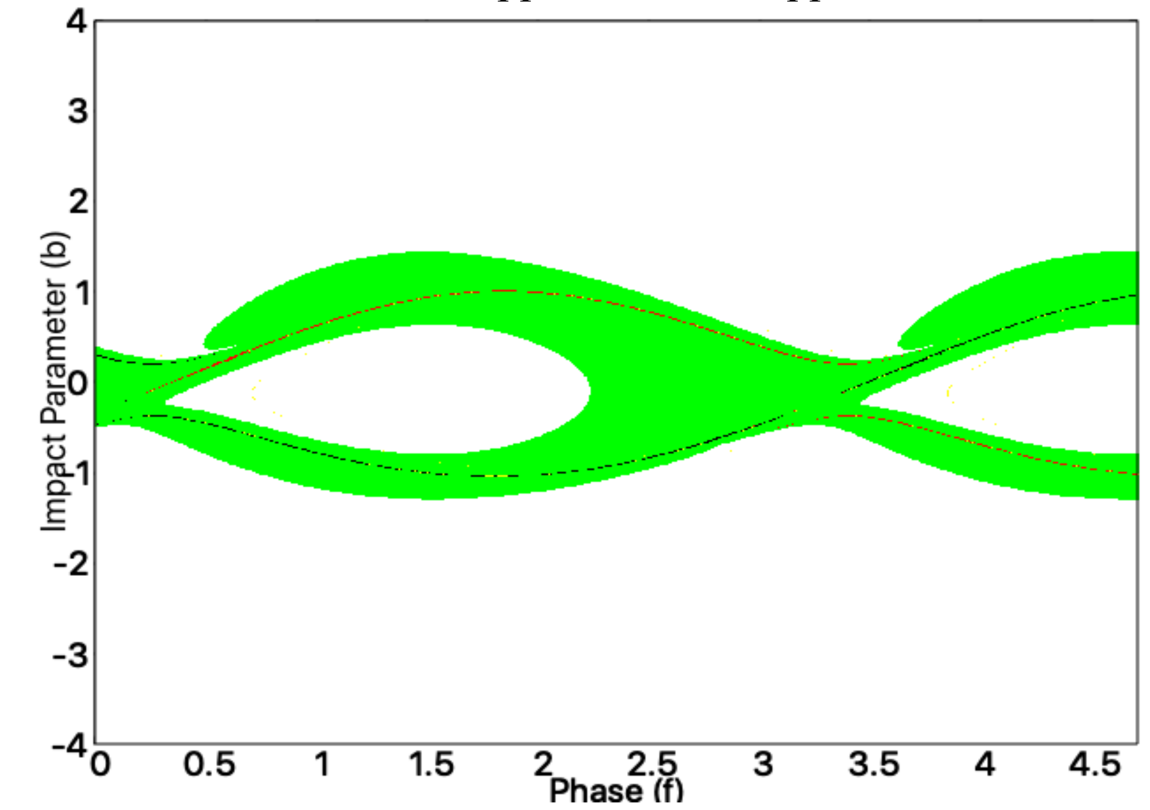

Figure 3: Scattering outcomes for $v=8v_c$, $R_0=40$, and initial step size of 0.001.

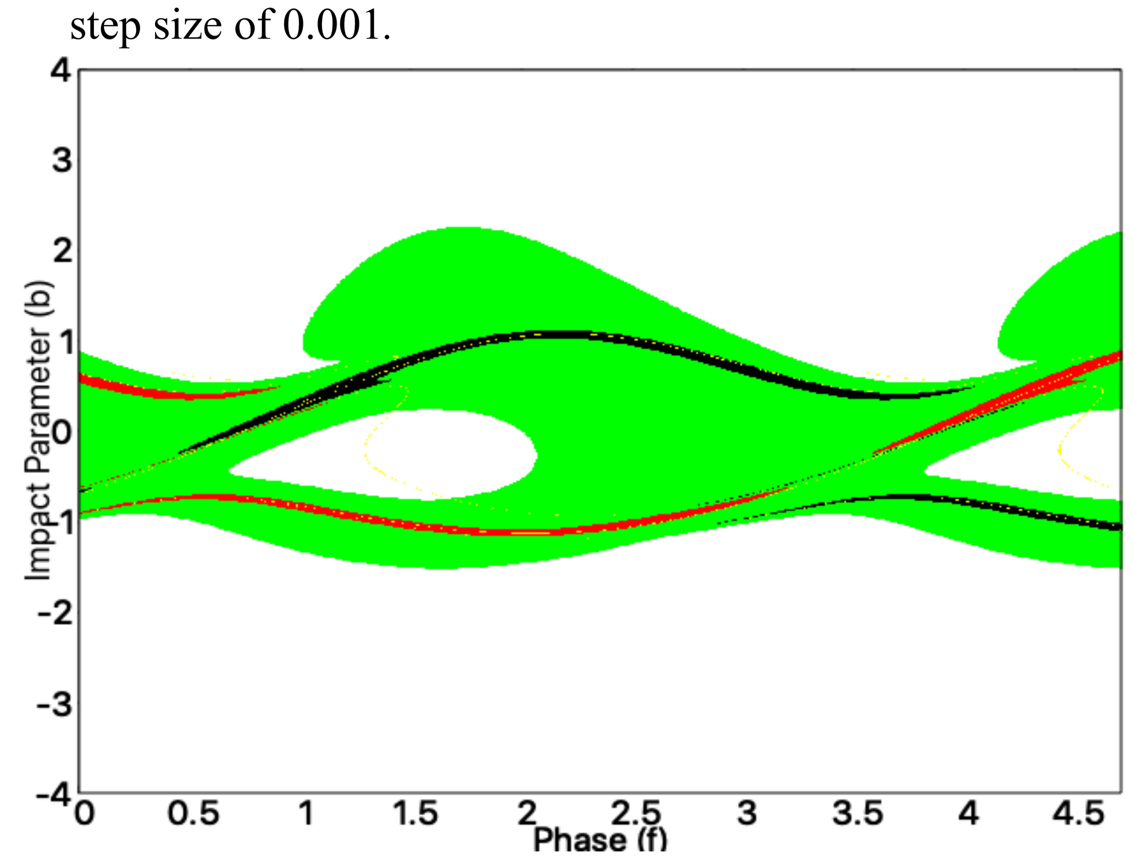

Figure 4: Scattering outcomes for $v=4v_c$, $R_0=40$, and initial step size of 0.001.

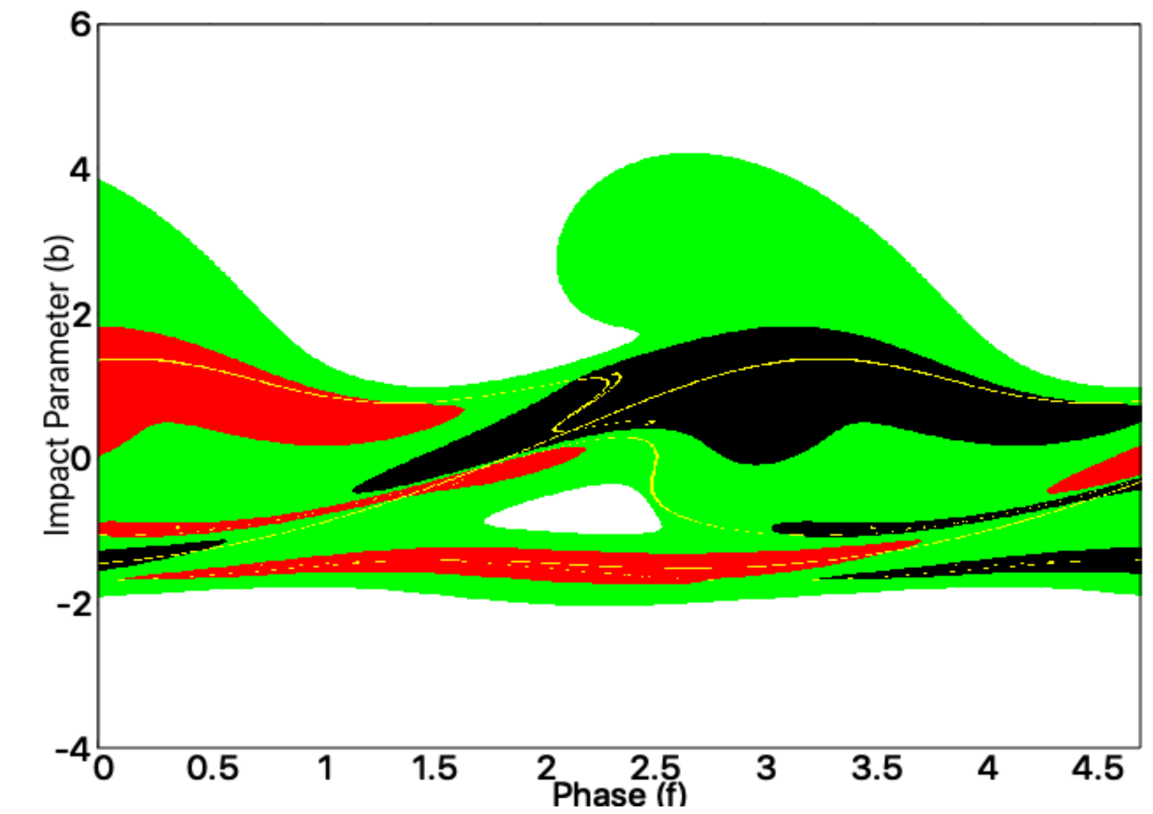

Figure 5: Scattering outcomes for $v=2v_c$, $R_0=40$, and initial step size of 0.001.

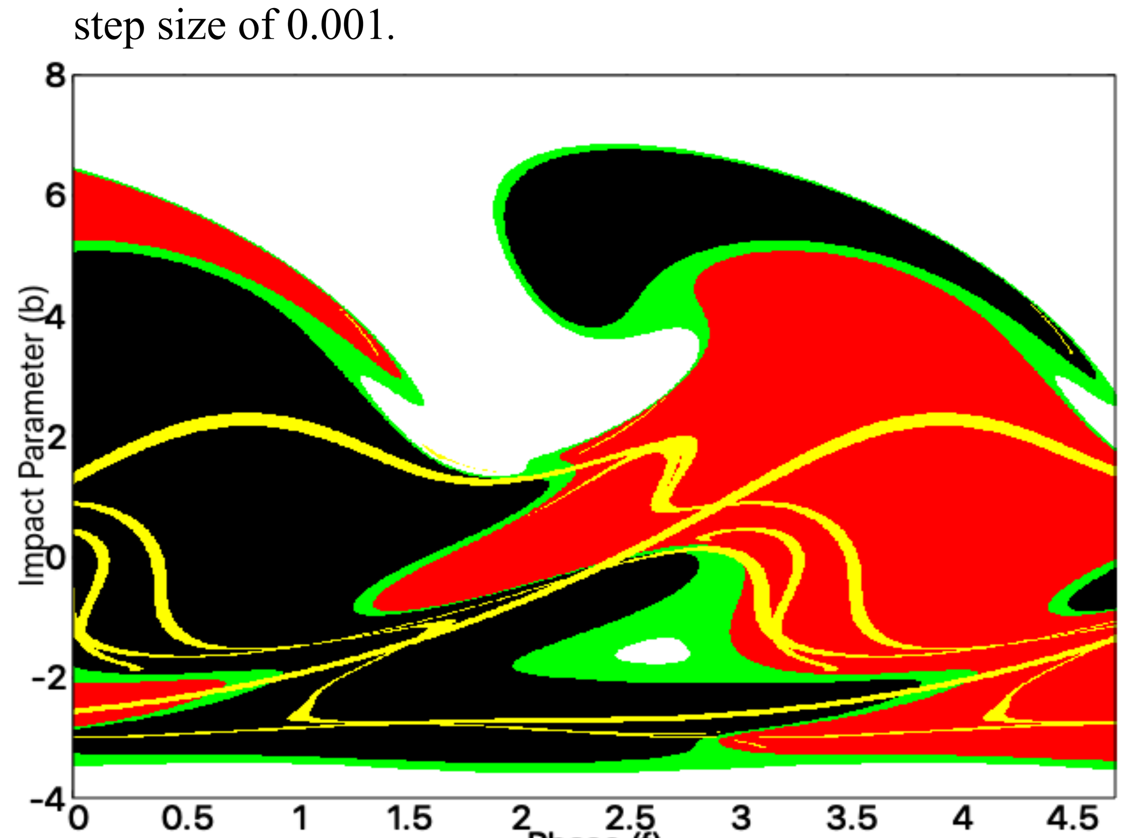

Figure 6: Scattering outcomes for $v=v_c$, $R_0=40$, and initial step size of 0.001.

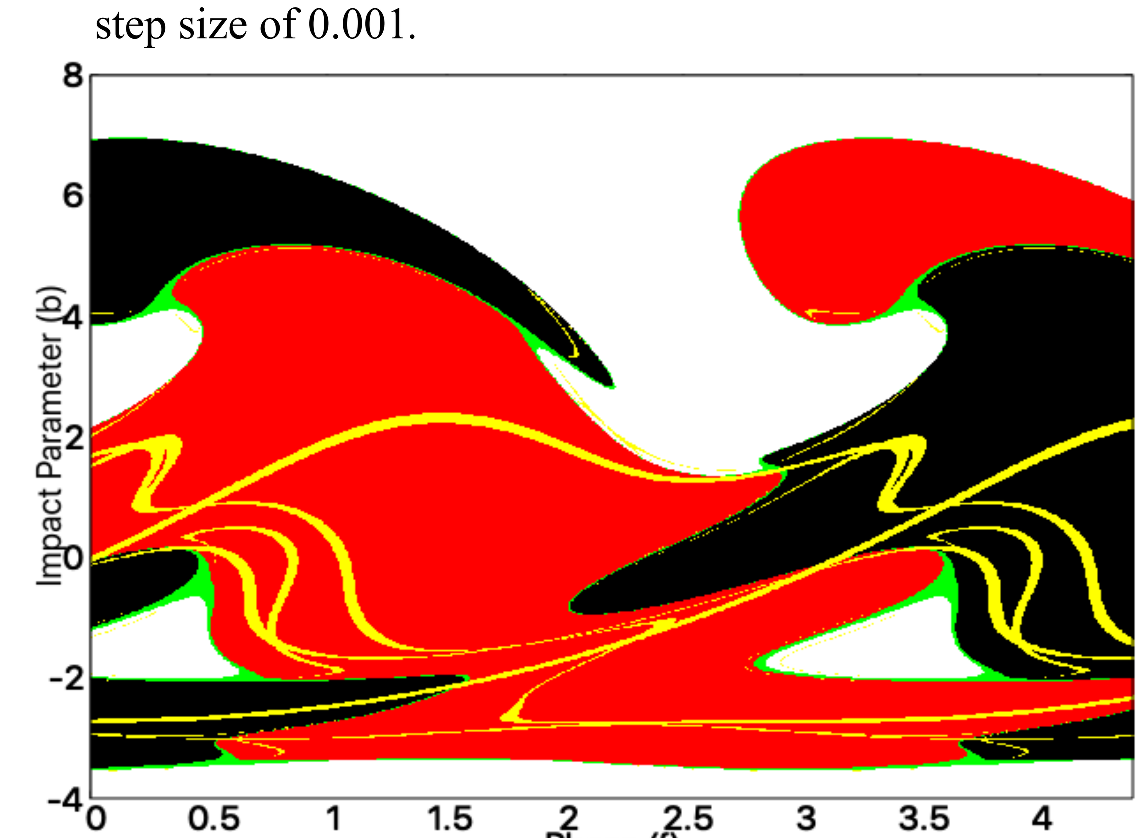

Figure 7: Scattering outcomes for $v=v_c$, $R_0=400$, and initial step size of 0.001.

Action Contour Maps

Although the literature on contour maps of action calculations is lacking, what can be currently said is that the radial and angular action maps have similar characteristics as seen in figures 8 and 9 in the case $v=16v_c$.

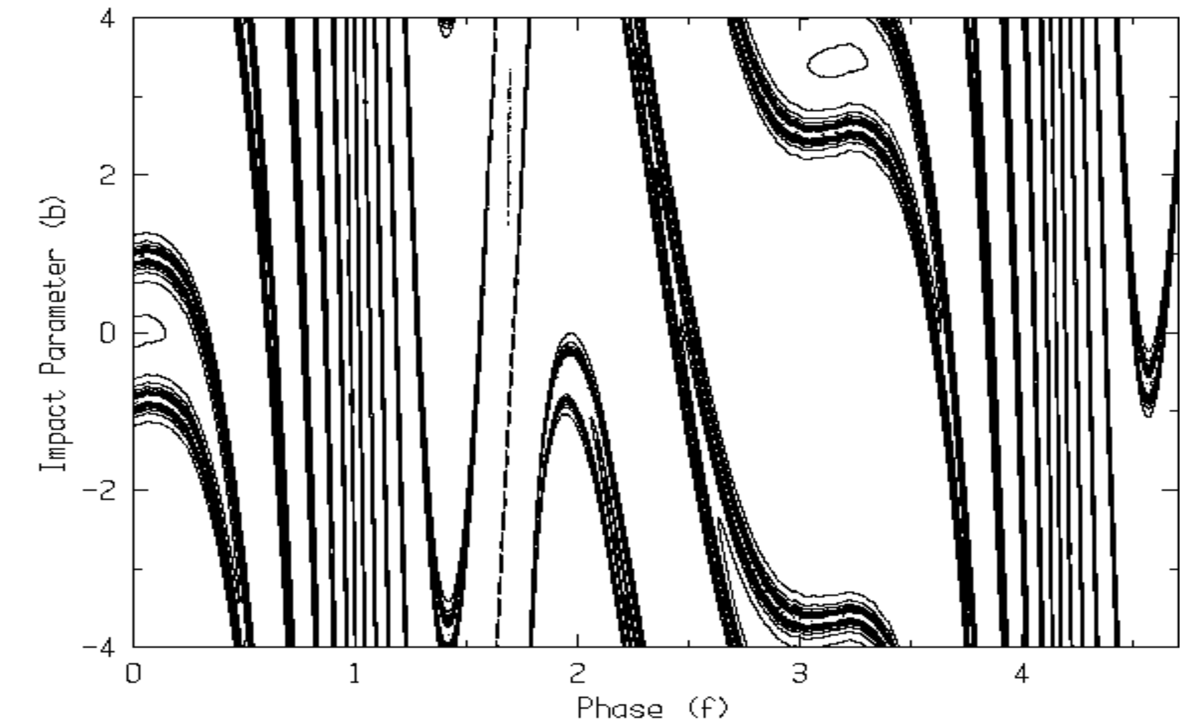

Figure 8: The radial action contour map for $v=16v_c$.

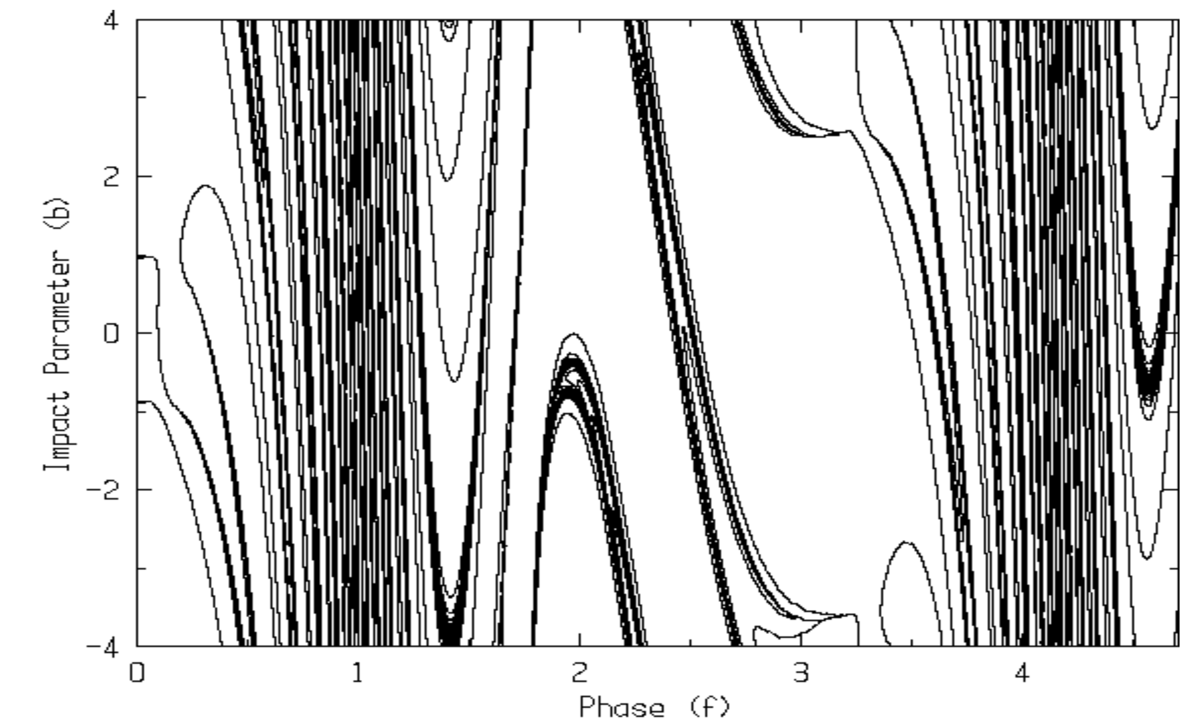

Figure 9: Rotational angular action contour map for $v=16v_c$.

Future Work

1. The program will be adjusted to better approximate the trajectory of field star coming from infinity to remove dissociation events in figure 7 and 8.
2. Further analysis of contour plots of final actions is necessary before any general statements can be made about them.
3. Currently, the trajectory program cannot deal with head-on or near head-on collisions between two of the masses. Morgan Long is working on improvements using the Levi-Civita transformation, which he discusses in his own research poster.

References

[H83] Hut, P. (1983). The topology of three-body scattering. The Astronomical Journal, 88, 1549–1559. https://doi.org/10.1086/113445

[HB83] Hut, P., & Bahcall, J. N. (1983). Binary-single star scattering. I - Numerical experiments for equal masses. The Astrophysical Journal, 268, 319–341. https://doi.org/10.1086/160956

[L08] Larner, A. (2008). Comparison of Molecular and Gravitational Three-Body Collisions (thesis).

Acknowledgements

I thank the College of Integrated Sciences for hosting this summer program and Wesleyan Mathematics and Science Scholars for providing grant funds for the experience.