Thermalization Process in a Multimode Nonlinear Photonic Network

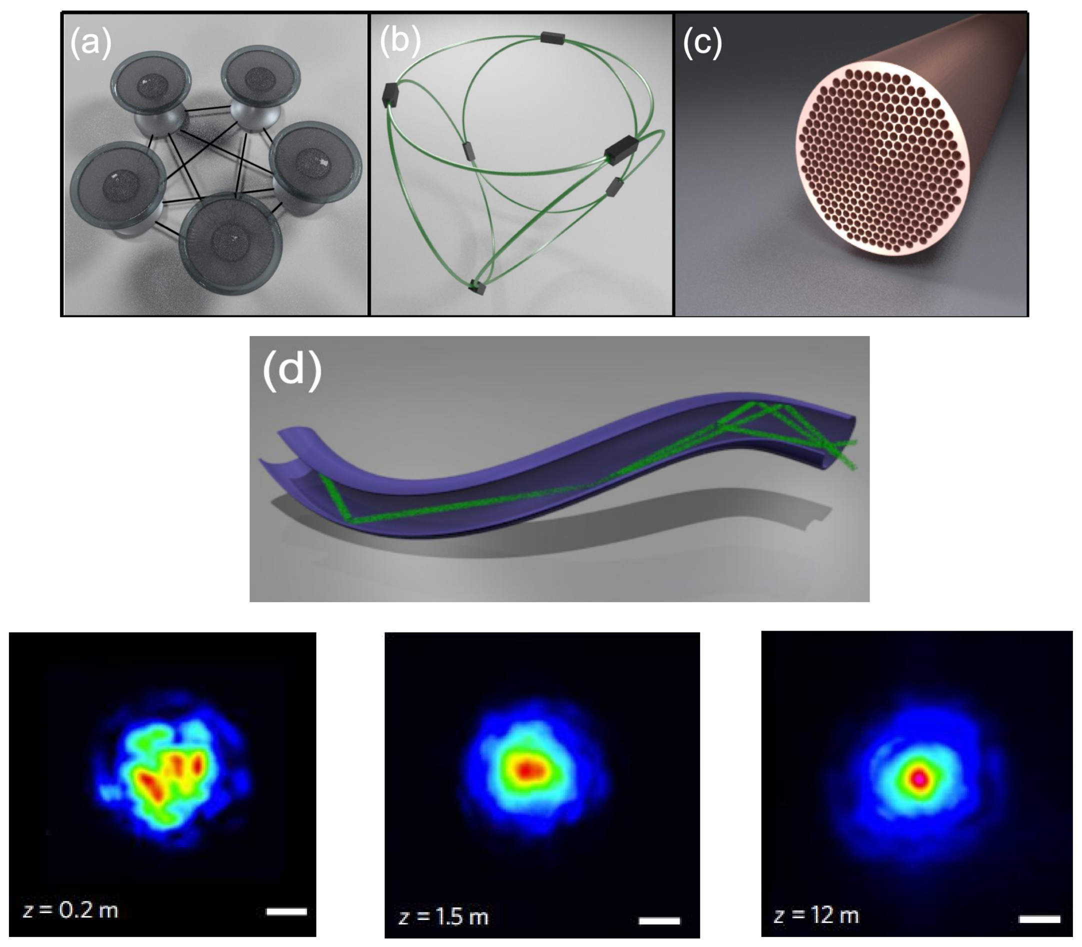
Cheng Shi, Lucas Fernández, Alba Ramos and Tsampikos Kottos
Wave Transport in Complex Systems Group, Department of Physics, Wesleyan University, Middletown, CT 06459 USA
Supported by College of Integrative Sciences, Wesleyan University.

Abstract

The thermalization process and the nature of the equilibrium state of an optical beam launched in a multimode nonlinear photonic network is a problem with far-reaching technological applications. Examples range from the design of high-power multimode sources, to beam self-cleaning (condensation) effects in fiber optics. The first step towards this analysis is to establish a reliable kinetic equation that will allow us to derive the thermodynamical properties of such systems. The next step is an in-depth understanding of the spectral properties of the underlying linear system (assuming weak nonlinearities).

In our work, we address both these questions, using simple one-dimensional networks with finite number of nodes and band-gap structures. Surprisingly, the finite degrees of freedom, associated with a finite number of network nodes, might give rise to negative temperatures (non-Gibbsian statistics) while under certain initial conditions, a two-phase thermalization occurs.

Motivation



Theoretical Modeling of Photonic Networks

$$i\frac{d\psi_l}{dt}=-\sum_j J_{lj}\psi_j+\chi|\psi_n|^2\psi_n,\qquad l=1,\dots,N$$

ψ_l: amplitudes in nodal space
J_{lj}: connectivity of nodes

$$\psi_l(t)=\sum_\alpha C_\alpha(t)f_\alpha(l)$$

$$i\frac{dC_\alpha}{dt}=\varepsilon_\alpha C_\alpha(t)+\chi\sum_{\beta,\lambda,\delta}\Gamma_{\alpha\beta\gamma\delta}C^*_\beta(t)C_\gamma(t)C_\delta(t)$$

C_α: amplitudes in modal space
ε_α: eigenvalues of the connectivity matrix

$$\Gamma_{\alpha\beta\gamma\delta}=\sum_{l=1}^{N}f_\alpha^*(l)f_\beta^*(l)f_\gamma(l)f_\delta(l)$$

Kinetic Equation

$$i\frac{dC_\alpha}{dt}=\varepsilon_\alpha C_\alpha(t)+\chi\sum_{\beta,\lambda,\delta}\Gamma_{\alpha\beta\gamma\delta}C^*_\beta(t)C_\gamma(t)C_\delta(t)$$

$C_\alpha(t)=A_\alpha(t)e^{-i\varepsilon_\alpha t}$

$$\frac{dA_\alpha}{dt}=-i\chi\sum_{\beta,\lambda,\delta}\Gamma_{\alpha\beta\gamma\delta}A^*_\beta A_\gamma A_\delta e^{i(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma-\varepsilon_\delta)t}$$

After the first iteration

$$A_\alpha(t+\Delta t)=A_\alpha(t)-i\chi\sum_{\beta,\gamma,\delta}\Gamma_{\alpha\beta\gamma\delta}A^*_\beta(t)A_\gamma(t)A_\delta(t)\int_t^{t+\Delta t}e^{i(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma-\varepsilon_\delta)\tau}d\tau$$

After the second iteration and random phase approximation

$$\frac{dI_\alpha}{dt}=4\pi\chi^2\sum_{\substack{(\alpha,\beta)\neq(\gamma,\delta)\\(\alpha,\beta)\neq(\delta,\gamma)}}|\Gamma_{\alpha\beta\gamma\delta}|^2(I_\alpha I_\gamma I_\delta+I_\beta I_\gamma I_\delta-I_\alpha I_\beta I_\gamma-I_\alpha I_\beta I_\delta)\delta(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma-\varepsilon_\delta)$$

, where $I_\alpha=|A_\alpha|^2=|C_\alpha|^2$

Equilibrium Distribution

$$\frac{dI_\alpha}{dt}=\sum_{\beta=1}^{k}\sum_{\gamma=1}^{k}\sum_{\delta=1}^{k}4\pi\chi^2(|\Gamma_{\alpha\beta\gamma\delta}|^2 I_\alpha I_\beta I_\gamma I_\delta(\frac{1}{I_\alpha}+\frac{1}{I_\beta}-\frac{1}{I_\gamma}-\frac{1}{I_\delta}))\delta(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma-\varepsilon_\delta)=0$$

$$\Leftrightarrow(\frac{1}{I_\alpha}+\frac{1}{I_\beta}-\frac{1}{I_\gamma}-\frac{1}{I_\delta})\delta(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma-\varepsilon_\delta)=0$$

An equilibrium solution: Rayleigh-Jeans distribution

$$I_\alpha=\frac{1}{\beta(\varepsilon_\alpha-\mu)}$$

β: inverse temperature
μ: chemical potential

$$(\frac{1}{I_\alpha}+\frac{1}{I_\beta}-\frac{1}{I_\gamma}-\frac{1}{I_\delta})\delta(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma-\varepsilon_\delta)$$

$$=\beta\varepsilon_\alpha+\beta\mu+\beta\varepsilon_\beta+\beta\mu-(\beta\varepsilon_\gamma+\beta\mu+\beta\varepsilon_\delta+\beta\mu)\delta(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma-\varepsilon_\delta)$$

$$=\beta(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma-\varepsilon_\delta)=0\Leftrightarrow\frac{dI_\alpha}{dt}=0$$

Relaxation Rates

Assuming all modes except for α are prepared in equilibrium, the relaxation rate of mode α will be:

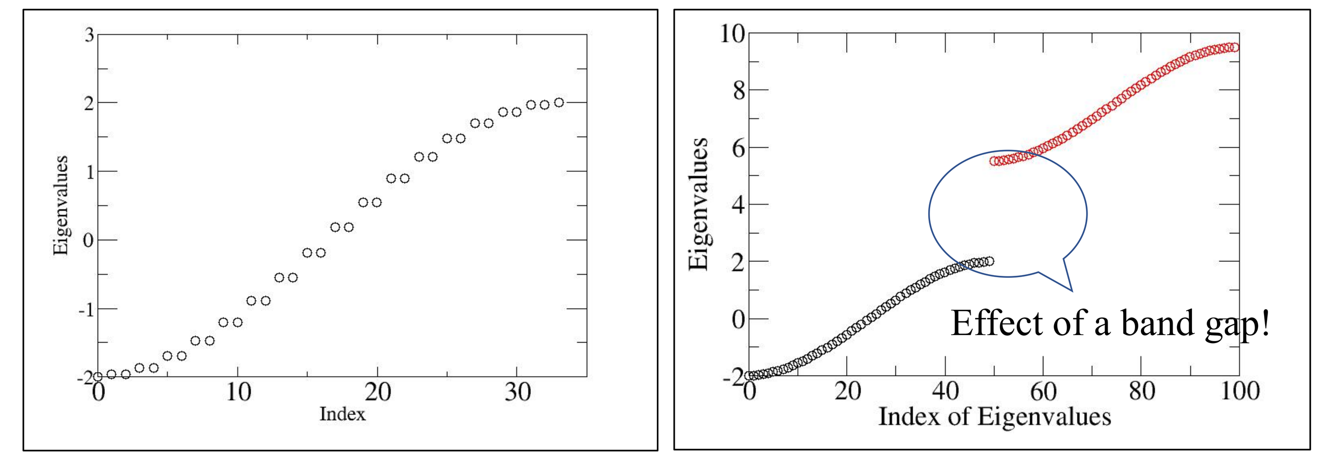
$$R_\alpha=\chi^2\sum_{\substack{(\alpha,\beta)\neq(\gamma,\delta)\\(\alpha,\beta)\neq(\delta,\gamma)}}|\Gamma_{\alpha\beta\gamma\delta}|^2(I_\beta I_\delta+I_\beta I_\gamma-I_\gamma I_\delta)\delta(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma-\varepsilon_\delta)$$

The time evolution for mode α:

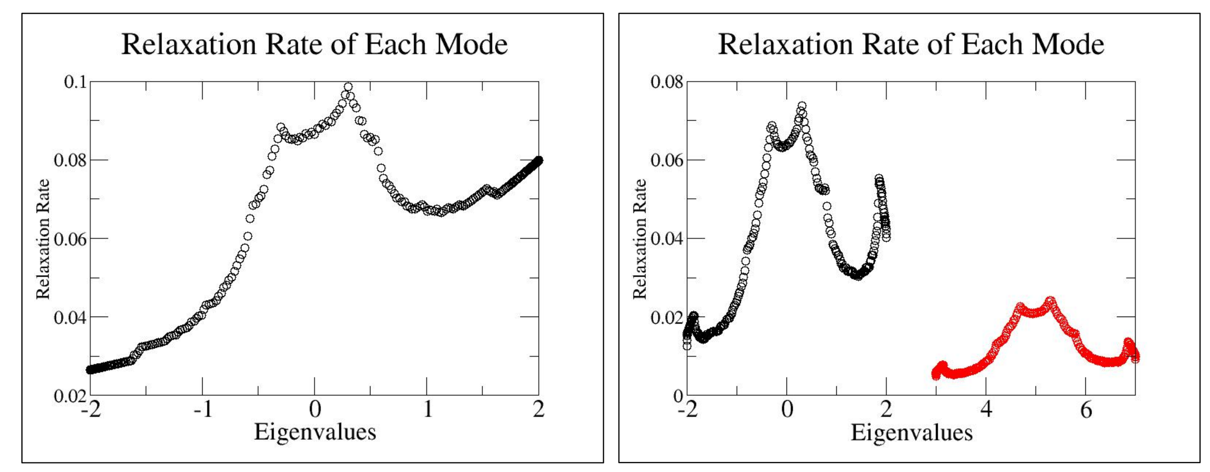
$$I_\alpha(t)=\frac{T}{\varepsilon_\alpha-\mu}+\left(I_\alpha(0)-\frac{T}{\varepsilon_\alpha-\mu}\right)e^{-R_\alpha t}$$

Numerical Results on Two Different Systems

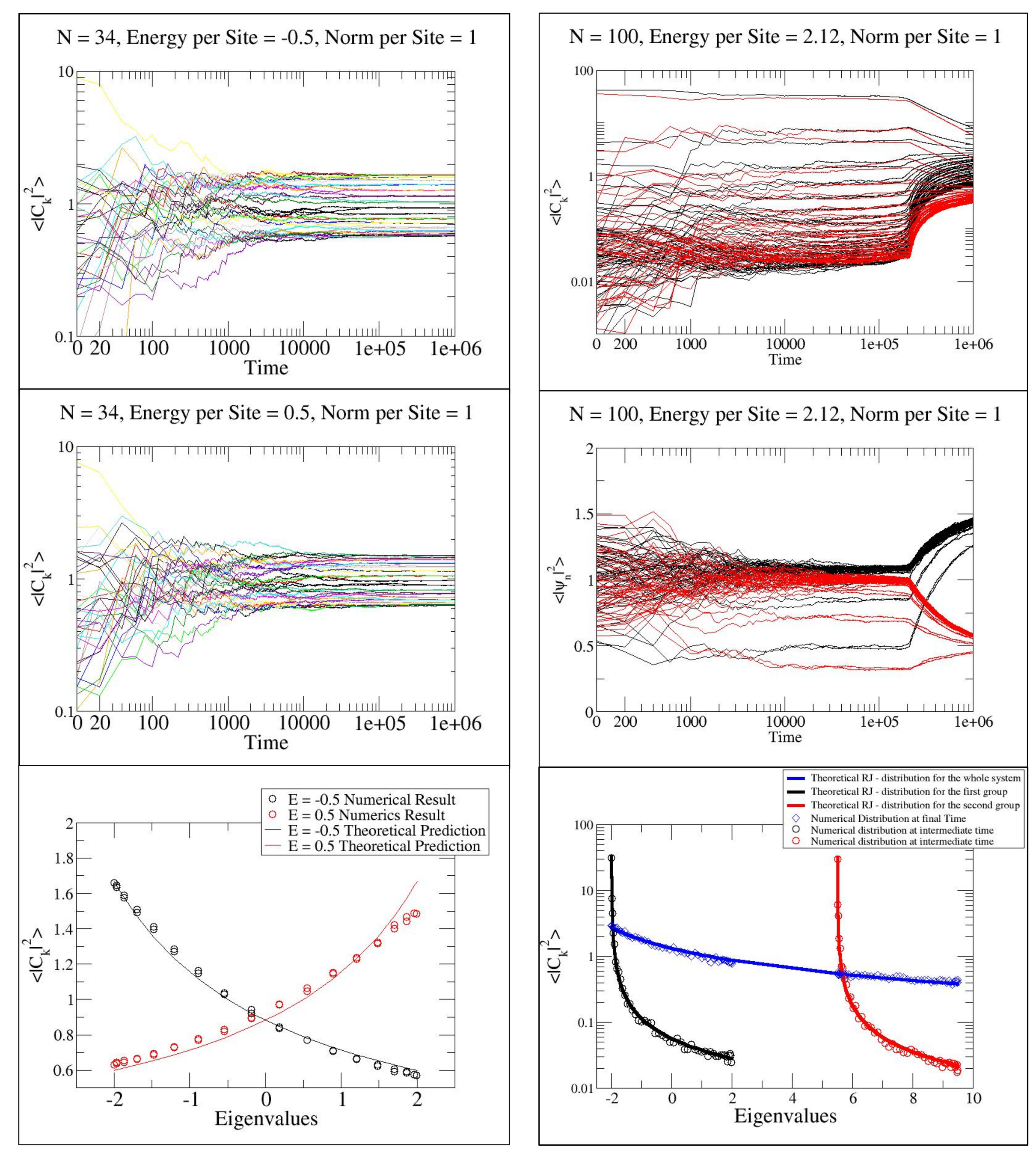
Spectrum



Relaxation Rates



Thermalization



References

[1] A.Ramos, L. Fernandez, T.Kottos, B.Shapiro, https://arxiv.org/abs/1912.10569
[2] Krupa, K., Tonello, A., Shalaby, B. et al. Spatial beam self-cleaning in multimode fibres. Nature Photon 11, 237–241 (2017).
[3] D.M. Basko, Phys. Rev. E 89, 022921 (2014).
[4] Nazarenko, Sergey. Wave Turbulence. Springer, 2011.